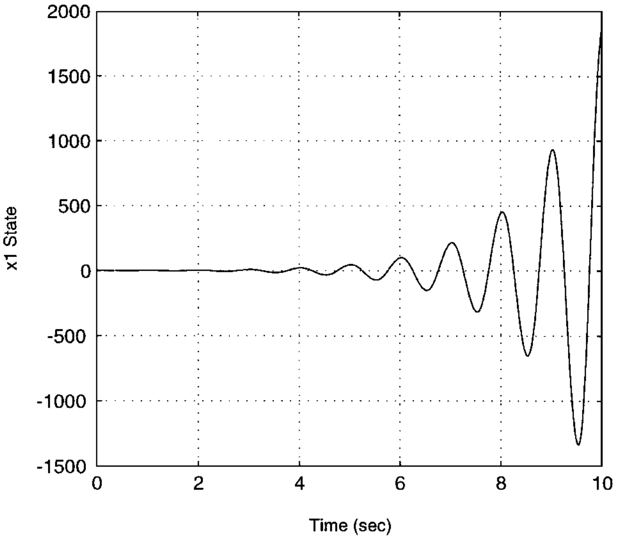

Fig. 1 Step response of the time-varying commutative system ($\bar{G} = \bar{K} =$ [1.2699 0]).

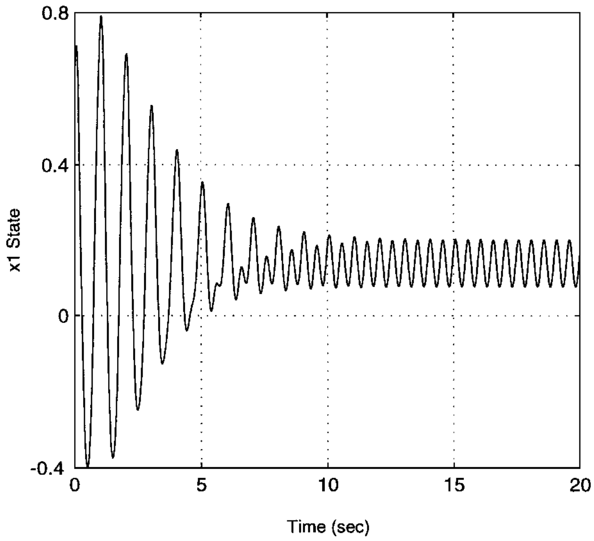

Fig. 2 Step response of the time-varying commutative system ($\bar{G} = \bar{K} =$ [3.7699 0]).

Figures 1 and 2 show that the time response of the system depends on the value of g_1 and k_1, respectively.

V. Conclusion

This Note is devoted to the design of the time-varying controller and estimator of the linear periodic time-varying system. New stability conditions have been developed based on Singh and Joseph's algorithm. The effectiveness of the result was demonstrated by numerical example where the controller and estimator design for the commutative time-varying system has been successfully achieved and the stability of the whole closed-loop system with different values of gain matrices considered.

Acknowledgment

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Adaptive Control of Free-Flying Space Robot with Position/Attitude Control System

Kei Senda,* Hideyuki Nagaoka,† and Yoshisada Murotsu‡

Osaka Prefecture University, Osaka 599-8531, Japan

Introduction

SEVERAL investigators have studied the free-flying space robot consisting of a satellite vehicle and manipulators.[1–4] All of the studies have discussed only the case when the manipulators do not handle any objects, or the payloads are known without uncertainty. Robust stability of those controllers has not been studied against the kinematical and dynamical variation. The performance becomes worse when the manipulators handle payloads with uncertain inertia parameters.[5] Murotsu et al. have proposed two parameter identification methods,[5] where a two-stage procedure is necessary, i.e., a controller must use the estimated parameters after the identification. Slotine and Li[6] proposed an adaptive control for the unknown payload manipulation on the ground, but it cannot be applied directly to space robots. Yamamoto et al.[7] and Iwata et al.[8] proposed adaptive controllers for space robots. The former is applicable under the translational and angular momentum conservation whereas the latter does not discuss the stability. Based on Slotine's method, this Note proposes a dynamics-based adaptive controller for the space robot with a position/attitude control system when the robot manipulates a payload.

Equations of Motion

In general, the equations of motion of a space robot can be derived as

$$\boldsymbol{M}\dot{\boldsymbol{u}} + \boldsymbol{h} = \boldsymbol{n} \tag{1}$$

where $\boldsymbol{u} = (\boldsymbol{v}_0^T \boldsymbol{\omega}_0^T \dot{\boldsymbol{\theta}}^T)^T$ and $\boldsymbol{n} = (\boldsymbol{f}_0^T \boldsymbol{\tau}_0^T \boldsymbol{\tau}_\theta^T)^T$. $\boldsymbol{M}$ is an inertia matrix, and $\boldsymbol{h}$ denotes the centrifugal and Coriolis forces. The variables $\boldsymbol{v}_0$ and $\boldsymbol{\omega}_0$ are the translational and the angular velocities of the satellite, respectively, and $\boldsymbol{\theta}$ is the vector of joint positions of the manipulators. The generalized forces $\boldsymbol{f}_0$ and $\boldsymbol{\tau}_0$ represent the translational and the rotational forces applied to the satellite, respectively. $\boldsymbol{\tau}_\theta$ denotes the joint forces. Formally the same equation as Eq. (1) is obtained by the Newton–Euler method and the Lagrangian method, though this study formulates the equations for numerical simulations by Kane's method in the same manner as Yamada et al.[9] If the satellite attitude variable $\boldsymbol{\phi}_0$ satisfies $\boldsymbol{\omega}_0 = \dot{\boldsymbol{\phi}}_0$, Eq. (1) can be rearranged as follows where $\dot{\boldsymbol{q}} = \boldsymbol{u}$:

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{h} = \boldsymbol{n} \tag{2}$$

The unknown parameters $\boldsymbol{\beta}$ to be estimated are the inertia parameters of the combined body of the hand and the payload manipulated by the robot. The left side of Eq. (2) can be linearized with respect to $\boldsymbol{\beta}$ as

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} = \boldsymbol{P}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})\boldsymbol{\beta} + \boldsymbol{Q} \tag{3}$$

where $\boldsymbol{P}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$ called the regressor is a matrix function of $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$, and $\ddot{\boldsymbol{q}}$. The first $\dot{\boldsymbol{q}}$ in $\boldsymbol{P}$ is the vector of independent variables in $\boldsymbol{C}$, and

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*Associate Professor, Department of Aerospace Engineering, Sakai. Senior Member AIAA.

†Graduate Student, Department of Aerospace Engineering, Sakai.

‡Professor, Department of Aerospace Engineering, Sakai. Senior Member AIAA.

the second one is premultiplied by $\boldsymbol{C}$. $\boldsymbol{Q}$ is a matrix function of the known parameters and the observable state variables. Substituting $\hat{\beta}$ into Eq. (3) produces a similar equation, and subtracting Eq. (3) from the obtained equation yields

$$\tilde{M}\ddot{q} + \tilde{C}\dot{q} = P(q, \dot{q}, \dot{q}, \ddot{q})\tilde{\beta} \tag{4}$$

where $\tilde{M} = \hat{M} - M$, $\tilde{C} = \hat{C} - C$, $\tilde{\beta} = \hat{\beta} - \beta$, and $\hat{M}$ and $\hat{C}$ represent M and C computed by substituting $\hat{\beta}$.

Adaptive Control

Baseline Adaptive Control

A controller and a Lyapunov function are taken as

$$n = \hat{M}\ddot{q}_r + \hat{C}\dot{q}_r - K_D\dot{\tilde{q}} \tag{5}$$

$$V = \tfrac{1}{2}(\dot{\tilde{q}}^T M\dot{\tilde{q}} + \tilde{\beta}^T \Gamma \tilde{\beta}) \tag{6}$$

where $\dot{\tilde{q}} \overset{\Delta}{=} \dot{q} - \dot{q}_r$ for a reference input q_r corresponding to q. Time derivative of V yields

$$\begin{aligned} \dot{V} &= \dot{\tilde{q}}^T M\ddot{\tilde{q}} + \tfrac{1}{2}\dot{\tilde{q}}^T \dot{M}\dot{\tilde{q}} + \tilde{\beta}^T \Gamma \dot{\tilde{\beta}} \\ &= \dot{\tilde{q}}^T \left[n + \tfrac{1}{2}(\dot{M} - 2C)\dot{\tilde{q}} - C\dot{q}_r - M\ddot{q}_r\right] + \tilde{\beta}^T \Gamma \dot{\tilde{\beta}} \\ &= \dot{\tilde{q}}^T [n - M\ddot{q}_r - C\dot{q}_r] + \tilde{\beta}^T \Gamma \dot{\tilde{\beta}} \qquad (7) \\ &= -\dot{\tilde{q}}^T K_D \dot{\tilde{q}} + \tilde{\beta}^T \left[\Gamma \dot{\tilde{\beta}} + P^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\dot{\tilde{q}}\right] \qquad (8) \end{aligned}$$

The skew symmetry of $\dot{M} - 2C$ is used in the rearrangement of Eq. (7). Equations (4) and (5) are used for obtaining Eq. (8). Hence an adaptation law

$$\dot{\hat{\beta}} = -\Gamma^{-1} P^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\dot{\tilde{q}} \tag{9}$$

leads to

$$\dot{V} = -\dot{\tilde{q}}^T K_D \dot{\tilde{q}} \leq 0 \tag{10}$$

because $\dot{\tilde{\beta}} = \dot{\hat{\beta}}$ is true. This inequality shows that $\dot{\tilde{q}} \to 0$ as $t \to \infty$. One obtains $\dot{\tilde{q}} = \dot{q}_e + \Lambda_q q_e \to 0$ as $t \to \infty$ if the reference input is $\dot{q}_r = \dot{q}_d - \Lambda_q q_e$ where q_d, $q_e = q - q_d$ and Λ_q are the desired joint variables, the joint variable errors, and the positive definite gain matrix, respectively. Because the steady-state solution of $\dot{q}_e + \Lambda_q q_e = 0$ is $q_e \equiv 0$, $q_e \to 0$ and $\dot{q}_e \to 0$ as $t \to \infty$. This result is true even though the convergence of $\hat{\beta}$ is not guaranteed.

Although the previous controller is for the joint variable control, a manipulation variable control is derived next in the case where both the hand and the satellite vehicle are controlled. The manipulation variables ψ are related to the variables q by Jacobian matrix J

$$\dot{\psi} = J\dot{q} \tag{11}$$

where $\dot{\psi} = (v_0^T \;\; \omega_0^T \;\; \dot{x}^T)^T$ and $\dot{x}$ is the velocity vector of the hand position and orientation. The $\dot{q}_r$ and $\ddot{q}_r$ are given by using the desired manipulation variables ψ_d and the positive definite gain matrix Λ:

$$\dot{q}_r = J^{-1}(\dot{\psi}_d - \Lambda\psi_e), \qquad \ddot{q}_r = J^{-1}(\ddot{\psi}_d - \Lambda\dot{\psi}_e - \dot{J}\dot{q}) \tag{12}$$

The following relation is then obtained from Eq. (10):

$$\dot{V} = -(\dot{\psi}_e + \Lambda\psi_e)^T J^{-T} K_D J^{-1}(\dot{\psi}_e + \Lambda\psi_e) \leq 0 \tag{13}$$

where $\psi_e \overset{\Delta}{=} \psi - \psi_d$ are the manipulation variable errors. The manipulation variable errors converge to zeroes because $\dot{\psi}_e + \Lambda\psi_e \to 0$ as $t \to \infty$ for the nonsingular J and $\dot{\psi}_e + \Lambda\psi_e = 0$ has the steady-state solution $\psi_e \equiv 0$. Therefore, $\psi_e \to 0$, and $\dot{\psi}_e \to 0$ as $t \to \infty$.

Consequently, the adaptive control can be achieved by $\dot{q}_r$ and $\ddot{q}_r$ of Eqs. (12), the control law of Eq. (5), and the adaptation law of Eq. (9).

Modification of Adaptive Control

The adaptive control can be modified for situations where the robot does not control the satellite position and/or the attitude. The needed modifications are as follows: 1) the current values of ψ are used as the desired values of the uncontrolled variables in ψ, and 2) the control inputs corresponding to the uncontrolled variables are set to zero. For example, $r_{0d} \equiv r_0$, $v_{0d} \equiv v_0$, $\dot{v}_{0d} \equiv \dot{v}_0$ are used for the desired values of the satellite position, the velocity, and the translational acceleration, respectively, when the satellite position is not controlled. The control input f_0 corresponding to the satellite position is set at zero. In Eqs. (12) Λ is selected as a block-diagonal matrix, which does not enable interaction between the error feedback of the controlled variables and the desired values of the uncontrolled variables, as follows:

$$\Lambda = \begin{bmatrix} \Lambda_v & 0 & 0 \\ 0 & & \\ 0 & & \Lambda_{\omega x} \end{bmatrix} \tag{14}$$

The preceding relations and Eq. (12) yield $\dot{\tilde{q}} = (0^T, \; \tilde{\omega}_0^T, \; \dot{\tilde{\theta}}^T)^T$. In the first term in Eq. (7), the part multiplied by zeroes in $\dot{\tilde{q}}^T$ cannot influence the subsequent discussion. Thus Eqs. (10) and (13) are true for any f_0 corresponding to the uncontrolled variables in n of Eq. (5). Hence the desired control can be realized by letting $f_0 \equiv 0$ when the satellite position is not controlled.

In the same manner a stable control can be realized when the satellite attitude is not controlled.

Numerical Simulations

The plant is supposed to be a hardware robot composed of a space vehicle and a three-link manipulator, which is a planar experimental system on the ground simulating a space robot.[10] The space vehicle has a control momentum gyro (CMG) and thrusters for the position/attitude control system. A mathematical model of the robot is used for the plant because the hardware system is under construction. The parameters of the robot model are listed in Table 1 where m_i, ℓ_i, a_i, and I_i indicate the mass of link i, the length of the link, the distance of the mass center from the revolution axis of the joint i, and the mass moment of inertia around its mass center, respectively. The links and joints are numbered from the base to the tip of the manipulator. Link 0 is the space vehicle and link 3 is the manipulator hand. In this case the following inertia parameters of link 3 and the payload are unknown:

$$\beta = (\beta_1, \beta_2, \beta_3, \beta_4)^T = (m_3, m_3 a_{3x}, m_3 a_{3y}, \; I_3)^T \tag{15}$$

where a_{3x} and a_{3y} are x and y coordinates of the mass center in the link 3 frame. In the following simulations the inertia parameters of link 3 without payload are used as the initial values of the estimated parameters $\hat{\beta}$.

The following simulations are for the most general case when the positions and orientations of both the manipulator hand and the satellite are controlled. At the initial time the space robot is stationary and moves for 3 s as shown in Fig. 1a. The seventh-order polynomial curves of time are used to interpolate the path of manipulation variables. The conditions of the experimental system, e.g., quantization, time delay, and so on, are contained in the numerical simulations.

A numerical simulation is carried out for resolved acceleration control (RAC) to compare with the adaptive control. The control law is

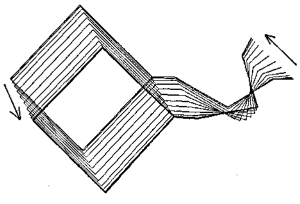
$$n = MJ^{-1}\{\ddot{\psi}_d - G_D\dot{\psi}_e - G_P\psi_e - \dot{J}\dot{q}\} + h \tag{16}$$

where G_P and G_D are the proportional and derivative feedback gains, respectively. The control input is computed by Yamada's algorithm.[4]

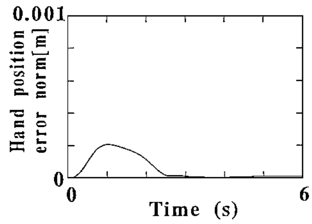
Figure 1 shows a) the robot motion, b) the error norm of the hand position, and c) the estimations of the unknown parameters, respectively. Figure 1c represents the ratios $(\hat{\beta}/\beta)$ of the estimated values

Table 1 Specification of space robot model

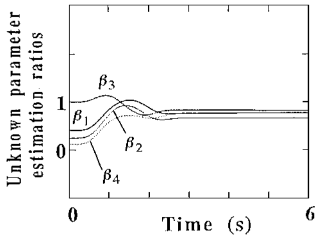
Link i	Mass m_i, kg	Length ℓ_i, m	Mass center a_i, m	Inertia I_i, kg/m^2
Link 0	46.94	0.459	0.000	1.413
Link 1	2.75	0.300	0.206	0.031
Link 2	1.62	0.260	0.138	0.017
Link 3 without payload	1.34	0.060	0.019	0.005
Link 3 with payload	3.29	0.245	0.019	0.042



a) Robot motion

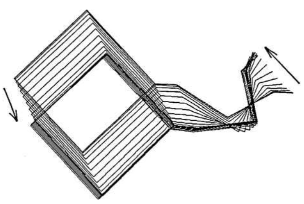


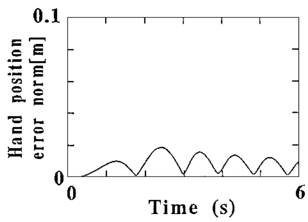
b) Hand position error



c) Parameter estimation

Fig. 1 Control result of adaptive control (K_D = diag[6.0 6.0 6.0 6.0 6.0 6.0], Λ = diag[160 160 20 160 160 20], Γ = diag[0.001 0.1 1.0 1.0]).





a) Robot motion b) Hand position error

Fig. 2 Control result of RAC (G_P = diag[160 160 120 160 160 120], G_D = diag[3.0 3.0 2.0 3.0 3.0 2.0]).

to the real values. Figure 2 is for the RAC. The figures show that the adaptive controller achieves better performance than the RAC. Hence, the adaptive controller is more effective than the RAC when the space robot manipulates an unknown payload. The estimated parameters tend to converge to the real values during the motion for 3 s as illustrated in Fig. 1c, whereas the convergence of the estimation is not guaranteed. The parameters converge to the real values when the robot repeats the same motion although the results are not shown here. No more simulation results are illustrated whereas the same results are obtained in some other cases where the position and/or the orientation of the satellite vehicle is not controlled.

Conclusion

The adaptive controller has been proposed for space robots manipulating payloads. The controller has been able to control all manipulation variables of the position and the orientation of the satellite and those of the hand. The small modification has allowed the controller to select any parts of the manipulation variables. The asymptotic stability of the closed-loop system was studied by Lyapunov's second method. The effectiveness of the controller has been examined by numerical simulations.

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Initial Adjoint-Variable Guess Technique and Its Application in Optimal Orbital Transfer

Hui Yan* and Hongxin Wu†
Chinese Academy of Space Technology,
100080 Beijing, People's Republic of China

Introduction

IT is well known that the main difficulty of the indirect method for trajectory optimization is the requirement for an educated guess of the initial adjoint variables. A number of papers have been proposed to deal with the guess. Perhaps the adjoint-control transformation method presented by Dixon[1] is the most efficient and attractive. This Note also investigates the adjoint-variable guess problem using the control variables. The approximate initial adjoint variables are obtained by solving equations in the neighborhood of the initial time in which the adjoint variables can be expressed by a first-order Taylor series expansion. Two numerical examples are presented for optimization of very low thrust trajectories.

Optimal Control Problems

Consider the state equations that are not explicit functions of time

$$\dot{x} = f[x(t), u(t)] \tag{1}$$

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*Postdoctoral Researcher, Beijing Institute of Control Engineering, P.O. Box 2729.

†Professor, Beijing Institute of Control Engineering.